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If A, B are square matrices of order n, and I_n is a corresponding unit matrix, then

- (a) $A(\text{adj.}A) = |A| I_n = (\text{adj } A) A$
- (b) $|\text{adj } A| = |A|^{n-1}$ (Thus A (adj A) is always a scalar matrix)
- (c) $\text{adj } (\text{adj.}A) = |A|^{n-2} A$
- (e) $|\text{adj } A, (\text{adj.}A)| = |A|^{\{1\}} \times \{(n-1)^{n-2}\} | \text{adj } (\text{adj.}A) | = |A|^{(n-1)^2}$
- (f) $\text{adj } (AB) = (\text{adj } B) (\text{adj } A)$
- (g) $\text{adj } (A^m) = (\text{adj } A)^m$,
- (h) $\text{adj } (kA) = \{k\}^{n-1} (\text{adj. } A)$, $k \in R$
- (i) $\text{adj } (I_n) = I_n$
- (j) $\text{adj } 0 = 0$
- (k) A is symmetric $\Rightarrow \text{adj } A$ is also symmetric
- (l) A is diagonal $\Rightarrow \text{adj } A$ is also diagonal
- (m) A is triangular $\Rightarrow \text{adj } A$ is also triangular
- (n) A is singular $\Rightarrow |\text{adj } A| = 0$

Types of Matrices

(i) **Symmetric Matrix:** A square matrix $A = [\{a_{ij}\}] = [a_{ij}]$ is called a symmetric matrix if $\{a_{ij}\} = \{a_{ji}\}$, $a_{ij} = a_{ji}$, for all i, j.

(ii) **Skew-Symmetric Matrix:** when $\{a_{ij}\} = -\{a_{ji}\}$ $a_{ij} = -a_{ji}$

(iii) **Hermitian and skew – Hermitian Matrix:** $A = \{A\}^{\theta}$ ($A = A^\theta$ (Hermitian matrix))

$\{A\}^{\theta} = -AA^\theta = -A$ (skew-Hermitian matrix)

(iv) **Orthogonal**

matrix: if $A\{A\}^T = \{I_n\} = \{A\}^T A$ $AA^T = I_n = A^TA$

(v) Idempotent matrix: if $\{ \{ A \}^{\wedge\{2\}} \} = AA_2 = A$

(vi) Involuntary matrix: if $\{ \{ A \}^{\wedge\{2\}} \} = I, \text{ or } \{ \{ A \}^{\wedge\{-1\}} \} = AA_2 = I \text{ or } A_{-1} = A$

(vii) Nilpotent matrix: if exists $p \in N$ such that $\{ \{ A \}^{\wedge\{P\}} \} = 0, A_P = 0$

Trace of matrix

(i) $\text{tr}(\lambda A) = \lambda \text{tr}(A)$

(ii) $\text{tr}(A+B) = \text{tr}(A) + \text{tr}(B)$

(iii) $\text{tr}(AB) = \text{tr}(BA)$

Matrix Transpose

(i) $\{ \{ \{ A \}^{\wedge\{T\}} \} \}^{\wedge\{T\}} = A$; (ii) $\{ \{ (A \pm B) \}^{\wedge\{T\}} \} = \{ \{ A \}^{\wedge\{T\}} \} \pm \{ \{ B \}^{\wedge\{T\}} \}$; (iii)

$\{ \{ (AB) \}^{\wedge\{T\}} \} = \{ \{ B \}^{\wedge\{T\}} \} \{ \{ A \}^{\wedge\{T\}} \}$ (i) $(AT)^T = A$ (ii) $(A \pm B)^T = A^T \pm B^T$ (iii) $(AB)^T = B^T A^T$

(iv) $\{ \{ (kA) \}^{\wedge\{T\}} \} = k \{ \{ \{ A \}^{\wedge\{T\}} \} \}^{\wedge\{T\}}$; (v)

$\{ \{ \{ A \}_1 \} \{ \{ A \}_2 \} \{ \{ A \}_3 \} \} \dots \{ \{ A \}_{n-1} \} \{ \{ A \}_n \} \}^{\wedge\{T\}} = A_n^{\wedge\{T\}} A_{n-1}^{\wedge\{T\}} \dots A_3^{\wedge\{T\}} A_2^{\wedge\{T\}} A_1^{\wedge\{T\}}$

(vi) $\{ \{ I \}^{\wedge\{T\}} \} = I$; (vii)
 $\text{tr}(A) = t(\{ \{ A \}^{\wedge\{T\}} \})$ (vi) $I^T = I$ (vii) $\text{tr}(A) = t(A^T)$